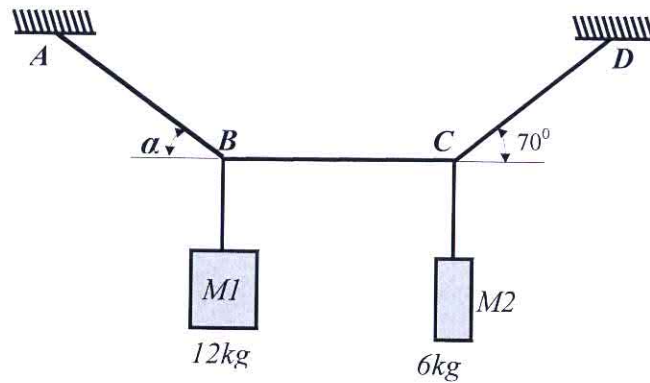




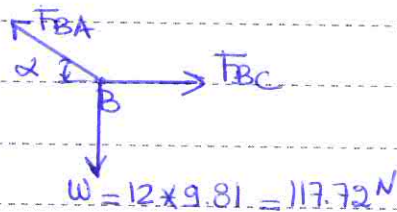
**Problem I:** (30 points)**Figure I**

The masses  $M_1=12\text{kg}$  and  $M_2=6\text{kg}$  are suspended by the cable system shown in Figure I.

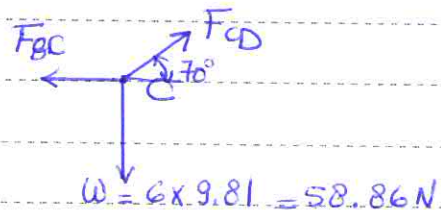
- Determine the angle  $\alpha$  and the tension in cables  $AB$ ,  $BC$  and  $CD$ .

Calculations and/or Diagrams:

F.B.D. at B



F.B.D. at C



Equilibrium at C:

$$+\rightarrow \sum F_x = 0 \Rightarrow -F_{BC} + F_{CD} \cos 70 = 0 \quad (1)$$

$$+\uparrow \sum F_y = 0 \Rightarrow F_{CD} \sin 70 - 58.86 = 0 \quad (2)$$

$$\text{From Eq. (2)} \Rightarrow \boxed{F_{CD} = 62.64 \text{ N}}$$

$$\text{From Eq. (1)} \Rightarrow -F_{BC} + 62.64 \cos 70 = 0 \Rightarrow \boxed{F_{BC} = 21.42 \text{ N}}$$

Calculations and/or Diagrams (cont'd):

Equilibrium at B

$$\begin{aligned} + \rightarrow \sum F_x = 0 &\Rightarrow -F_{BA} \cos \alpha + 21.42 \text{ N} = 0 \\ &\Rightarrow F_{BA} \cos \alpha = 21.42 \quad \text{--- (3)} \end{aligned}$$

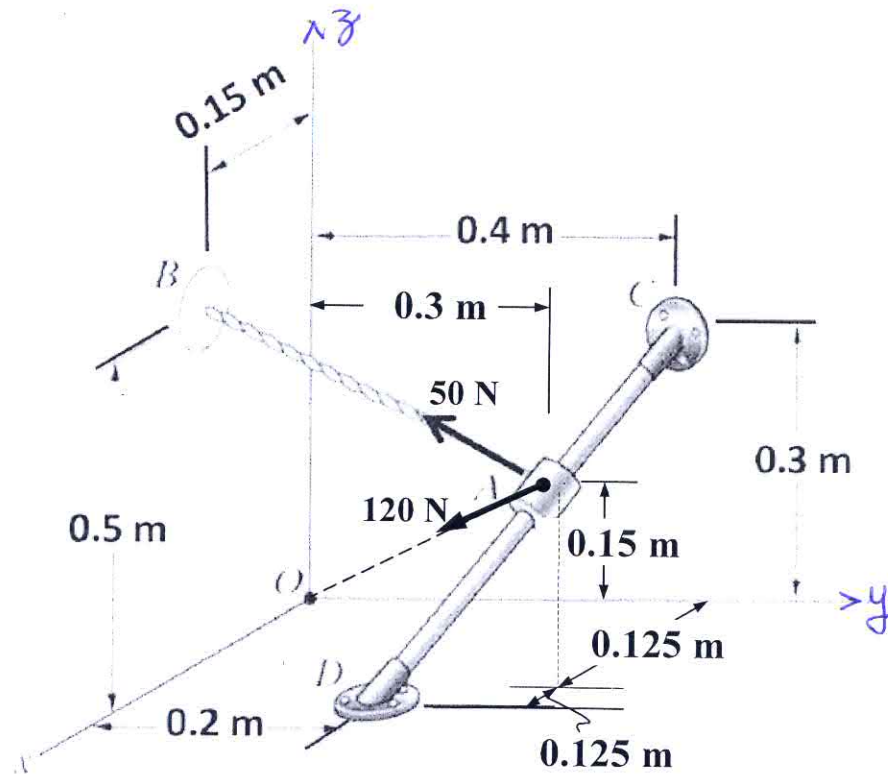
$$\begin{aligned} + \uparrow \sum F_y = 0 &\Rightarrow F_{BA} \sin \alpha - 117.72 = 0 \\ &\Rightarrow F_{BA} \sin \alpha = 117.72 \quad \text{--- (4)} \end{aligned}$$

$$\begin{aligned} \frac{\text{Eq. (4)}}{\text{Eq. (3)}} &\Rightarrow \frac{F_{BA} \sin \alpha = 117.72}{F_{BA} \cos \alpha = 21.42} \Rightarrow \tan \alpha = \frac{117.72}{21.42} \end{aligned}$$

$$\Rightarrow \boxed{\alpha = 79.7^\circ}$$

$$\text{From Eq. (3)} \Rightarrow F_{BA} \cos 79.7 = 21.42$$

$$\Rightarrow \boxed{F_{BA} = 119.8 \text{ N}}$$

**Problem II:** (35 points)**Figure II**

The rod CD is subjected to two forces as shown in Figure II.

1. Determine the magnitude and direction of the resultant force  $F_R$ . (18 points)
2. Determine the projected components of the resultant force  $F_R$  along line  $CD$  and perpendicular to the line  $CD$ . (10 points). *Express results in cartesian vector.*
3. Determine the angle between the two forces shown in Figure II. (7 points)

Calculations and/or Diagrams (cont'd):

1. Coordinates:  $O(0,0,0)$   $A(0.125, 0.3, 0.15)$   $B(0.15, 0, 0.5)$

$C(0, 0.4, 0.3)$   $D(0.25, 0.2, 0)$

Express forces as cartesian vector:

$$\vec{F}_{AB} = F_{AB} \vec{u}_{AB} = 50 \left\{ \frac{(0.15 - 0.125)\vec{i} + (0 - 0.3)\vec{j} + (0.5 - 0.15)\vec{k}}{\sqrt{(0.025)^2 + (-0.3)^2 + (0.35)^2}} \right\}$$

$$\vec{F}_{AB} = \{ 2.71\vec{i} - 32.47\vec{j} + 37.88\vec{k} \} \text{ N}$$

Calculations and/or Diagrams (cont'd):

$$\vec{F}_{AO} = F_{AO} \vec{u}_{AO} = 120 \left\{ \frac{-0.125\vec{i} + 0.3\vec{j} + 0.15\vec{k}}{\sqrt{(0.125)^2 + (0.3)^2 + (0.15)^2}} \right\}$$

$$\Rightarrow \vec{F}_{AO} = \{-41.9\vec{i} - 100.56\vec{j} - 50.28\vec{k}\}^N$$

Resultant force:

$$F_{Rx} = 2.71 - 41.9 = -39.19 \text{ N}$$

$$F_{Ry} = -32.47 - 100.56 = -133.03 \text{ N}$$

$$F_{Rz} = 37.88 - 50.28 = -12.4 \text{ N}$$

$$\vec{F}_R = \{-39.19\vec{i} + 133.03\vec{j} + 12.4\vec{k}\}^N$$

$$\text{Magnitude: } F_R = \sqrt{(-39.19)^2 + (-133.03)^2 + (-12.4)^2} = 139.24 \text{ N}$$

$$F_R = 139.24 \text{ N}$$

Direction

$$\cos \alpha = \frac{-39.19}{139.24}$$

$$\alpha = 106.35^\circ$$

$$\cos \beta = \frac{-133.03}{139.24}$$

$$\beta = 162.8^\circ$$

$$\cos \gamma = \frac{-12.4}{139.24}$$

$$\gamma = 95.1^\circ$$

$$2. \vec{u}_{CO} = \frac{0.25\vec{i} - 0.2\vec{j} - 0.3\vec{k}}{\sqrt{(0.25)^2 + (-0.2)^2 + (-0.3)^2}} = 0.569\vec{i} - 0.456\vec{j} - 0.683\vec{k}$$

$$\vec{F}_{R/CO} = F_R \cdot \vec{u}_{CO} = \{-39.19\vec{i} - 133.03\vec{j} - 12.4\vec{k}\} \cdot \{0.569\vec{i} - 0.456\vec{j} - 0.683\vec{k}\}$$

Calculations and/or Diagrams (cont'd):

$$\Rightarrow \boxed{F_{R//CD} = 46.83 \text{ N}}$$

$$F_{R\perp CD} = \sqrt{F_R^2 - F_{R//CD}^2} = \sqrt{(139.24)^2 - (46.83)^2} = 131.13 \text{ N}$$

$$\boxed{F_{R\perp CD} = 131.13 \text{ N}}$$

$$\vec{F}_{R//CD} = F_{R//CD} \vec{u}_{CD} = 46.83 \left\{ 0.569\vec{i} - 0.456\vec{j} - 0.683\vec{k} \right\}$$

$$\boxed{\vec{F}_{R//CD} = \left\{ 26.65\vec{i} - 21.35\vec{j} - 31.98\vec{k} \right\} \text{ N}}$$

$$\vec{F}_{R\perp CD} = F_{R\perp CD} \vec{u}_{\perp CD}$$

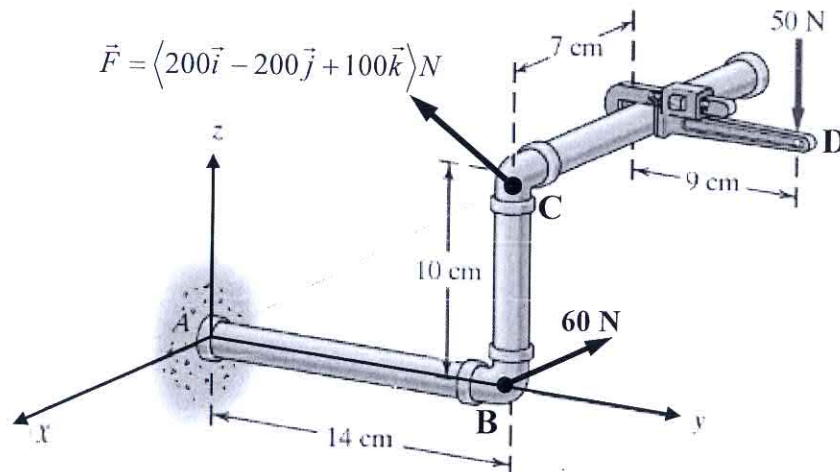
$$= 131.3 \left\{ 0.569\vec{i} - 0.456\vec{j} - 0.683\vec{k} \right\}$$

$$\boxed{\vec{F}_{R\perp CD} = \left\{ 74.71\vec{i} - 59.87\vec{j} - 89.68\vec{k} \right\} \text{ N}}$$

3.  $\vec{F}_{AB} \cdot \vec{F}_{AO} = F_{AB} F_{AO} \cos \theta$

$$\left\{ 2.71\vec{i} - 32.47\vec{j} + 37.88\vec{k} \right\} \cdot \left\{ -41.9\vec{i} - 100.56\vec{j} - 50.28\vec{k} \right\} = (50)(120) \cos \theta$$

$$\Rightarrow \boxed{\theta = 78^\circ}$$

**Problem III:** (35 points)**Figure III**

The rigid pipe system is subjected to the forces shown in Figure III.

1. Use vector approach, compute the moment from the three forces at the support A. (18 points).
2. Use scalar Approach, Re-compute the three components  $M_x$ ,  $M_y$ , and  $M_z$  at A due to the three forces, and compare with question 1. (7 points)
3. Determine the component of this moment about an axis extending between points A and C. Express the result as Cartesian vector. (10 points)

Calculations and/or Diagrams:

Coordinates:  $A(0,0,0)$   $B(0,14,0)$   $C(0,14,10)$   
 $D(-7,23,10)$

1. Vector Approach

$$\vec{r}_{AB} = \{0\vec{i} + 14\vec{j} + 0\vec{k}\} \text{ cm}$$

$$\vec{r}_{AC} = \{0\vec{i} + 14\vec{j} + 10\vec{k}\} \text{ cm}$$

$$\vec{r}_{AD} = \{-7\vec{i} + 23\vec{j} + 10\vec{k}\} \text{ cm}$$

$$\vec{F}_B = \{-60\vec{i} + 0\vec{j} + 0\vec{k}\} \text{ N}$$

Calculations and/or Diagrams (cont'd):

$$\vec{F}_D = \{0\vec{i} + 0\vec{j} - 50\vec{k}\}^N$$

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F}_B + \vec{r}_{AC} \times \vec{F} + \vec{r}_{AD} \times \vec{F}_D$$

$\vec{i}$	$\vec{j}$	$\vec{k}$		$\vec{i}$	$\vec{j}$	$\vec{k}$		$\vec{i}$	$\vec{j}$	$\vec{k}$
0	14	0	+	0	14	10	+	-7	23	10
-60	0	0		200	-200	100		0	0	-50

$$= \{0\vec{i} + 0\vec{j} + 840\vec{k}\} + \{2400\vec{i} + 2000\vec{j} - 2800\vec{k}\} + \{-1150\vec{i} + 350\vec{j} + 0\vec{k}\}$$

$$\vec{M}_A = \{2250\vec{i} + 1650\vec{j} - 1960\vec{k}\} \text{ N}\cdot\text{cm}$$

2. Scalar Approach:

$$M_x = 200(10) + 100(14) - 50(23) = 2250 \text{ N}\cdot\text{cm}$$

$$M_y = 200(10) - 50(7) = 1650 \text{ N}\cdot\text{cm}$$

$$M_z = 60(14) - 200(14) = -1960 \text{ N}\cdot\text{cm}$$

Same as Q1.

$$3. \mu_{AC} = \frac{0\vec{i} + 14\vec{j} + 10\vec{k}}{\sqrt{(0)^2 + (14)^2 + (10)^2}} = 0\vec{i} + 0.814\vec{j} + 0.581\vec{k}$$

$$M_{A//AC} = \vec{M}_A \cdot \mu_{AC} = \{2250\vec{i} + 1650\vec{j} - 1960\vec{k}\} \cdot \{0\vec{i} + 0.814\vec{j} + 0.581\vec{k}\}$$

$$M_{A//AC} = 204.34 \text{ N}\cdot\text{cm}$$



**EXTRA SHEET 1: Continued from page****Name:** \_\_\_\_\_**ID#:** \_\_\_\_\_

Calculations and/or Diagrams:

$$\begin{aligned}\vec{M}_{A/AC} &= M_{A/AC} \vec{u}_{AC} \\ &= 204.34 \{ 0\vec{i} + 0.814\vec{j} + 0.581\vec{k} \}\end{aligned}$$

$$\vec{M}_{A/AC} = \{ 0\vec{i} + 166.33\vec{j} + 118.72\vec{k} \} \text{ N}\cdot\text{cm}$$

